ENERGY IN S·H·M →

A body exhibiting simple harmonic motion has potential energy by virtue of its displacement from the mean position. By virtue of its velocity, it also possesses kinetic energy. During the oscillatory motion, these energies vary, but their sum remains constant, provided no dissipative forces are present. As expected for any motion under any conservative force, for a particle executing S.H.M, the total mechanical energy is independent of time.

Potential energy →

Let m denote the mass of a particle exhibiting simple harmonic motion and y denote the acceleration of the particle at any instant then by the characteristics of simple harmonic motion, we have

$$a = -\omega^2 y$$

where ω is the angular velocity of the particle, and thus ω^2 is a constant. At a given instant, the magnitude of the restoring force acting on the particle can be expressed as:

JUNE						2012
M	T	W	T	F	S	S
				1	2	3
4	5	6	7	8	9	10
11	12	13	14	15	16	17
18	19	20	21	22	23	24
25	26	27	28	29	30	

JULY						2012
M	T	W	T	F	S	S
30	31					1
2	3	4	5	6	7	8
9	10	11	12	13	14	15
16	17	18	19	20	21	22

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$F = \text{mass} \times \text{acceleration}$

$F = m \times -\omega^2 Y$

Now, considering the particle moves further by infenetesimally small displacement dY, then the work done against the force is expressed as

$$dW = (-F)\,dy = m\omega^2 Y\,dY \quad \text{—(1)}$$

Integrating eqn (1) between the limits $y = 0$ to $Y = y$, we get the total work done in displacing the particle through Y distance, that is

$$W = \int_0^Y m\omega^2 Y\,dY = m\omega^2\left[\frac{Y^2}{2}\right]_0^Y$$

$$= \frac{1}{2} m\omega^2 Y^2$$

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This work done on the particle is the potential energy U of the particle at that given instant

$$\boxed{U = \frac{1}{2} m\omega^2 Y^2}$$

—(1)

2012

W	T	F	S	S
2	3	4	5	6
9	10	11	12	13
16	17	18	19	20
23	24	25	26	27
30	31			

JUNE 2012

M	T	W	T	F	S	S
				1	2	3
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Kinetic Energy →

The velocity of a particle in simple harmonic motion is expressed as.

$$V = \omega\sqrt{A^2 - y^2}$$

Where A denotes the amplitude and y denotes the displacement of the particle. Hence, at any given instant, the kinetic energy of the particle is given by

$$K = \frac{1}{2} m v^2$$

$$K = \frac{1}{2} \omega m^2 (A^2 - y^2) \quad \text{— } 3$$

Total energy →

The total energy of the particle is obtained by adding eqn 2 and 3 and can be expressed as

$$E = U + K$$

$$E = \frac{1}{2} m \omega^2 y^2 + \frac{1}{2} m\omega^2 (A^2 - y)^2 \quad \text{— } 4$$

on simplification, the above equation can be written as

$$E = \frac{1}{2} m \omega^2 A^2 \quad \text{——} \quad (5)$$

where angular velocity of the particle is ω and A is the maximum displacement.

From eqn (5) it can be concluded that the total mechanical energy of the particle is free from the displacement y, which implies, total energy remains the same during the motion of the body.